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Multi-level Bayesian analyses for single- and multi-vehicle freeway crashes

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ABSTRACT

This study presents multi-level analyses for single- and multi-vehicle crashes on a mountainous freeway. Data from a 15-mile mountainous freeway section on I-70 were investigated. Both aggregate and disaggregate models for the two crash conditions were developed. Five years of crash data were used in the aggregate investigation, while the disaggregate models utilized one year of crash data along with real-time traffic and weather data. For the aggregate analyses, safety performance functions were developed for the purpose of revealing the contributing factors for each crash type. Two methodologies, a Bayesian bivariate Poisson-lognormal model and a Bayesian hierarchical Poisson model with correlated random effects, were estimated to simultaneously analyze the two crash conditions with consideration of possible correlations. Except for the factors related to geometric characteristics, two exposure parameters (annual average daily traffic and segment length) were included. Two different sets of significant explanatory and exposure variables were identified for the single-vehicle (SV) and multi-vehicle (MV) crashes. It was found that the Bayesian bivariate Poisson-lognormal model is superior to the Bayesian hierarchical Poisson model, the former with a substantially lower DIC and more significant variables. In addition to the aggregate analyses, microscopic real-time crash risk evaluation models were developed for the two crash conditions. Multi-level Bayesian logistic regression models were estimated with the random parameters accounting for seasonal variations, crash-unit-level diversity and segment-level random effects capturing unobserved heterogeneity caused by the geometric characteristics. The model results indicate that the effects of the selected variables on crash occurrence vary across seasons and crash units; and that geometric characteristic variables contribute to the segment variations: the more unobserved heterogeneity have been accounted, the better classification ability. Potential applications of the modeling results from both analysis approaches are discussed.

1. Introduction

Analyzing crash occurrence mechanisms and potential countermeasures have been extensively researched to develop aggregate safety performance functions (SPFs) using extensive methodologies (Lord and Mannering, 2010). Additionally, to identify crash-prone traffic statuses, disaggregate real-time crash risk evaluation models have been estimated. Aggregate analyses mainly focus on discovering the hazardous factors related to the frequency of total crashes, specific crash types or crash injury severity levels. Disaggregate studies benefit from reliable surveillance systems that provide detailed traffic and weather data for crashes. This information could help capture the micro–level influences of the hazardous factors that might lead to crash occurrence. Moreover, different data resources have been utilized in traffic safety studies, including traffic flow information (speed, volume, and lane occupancy), roadway geometric characteristics, and weather factors (visibility, precipitation).

This study presents both aggregate and disaggregate analyses for single-vehicle (SV) and multi-vehicle (MV) crashes on a mountainous freeway. Data from a 15-mile mountainous freeway section on I-70 in Colorado were utilized. The studied freeway section features a mountainous geometry (steep slopes up to 7%) and adverse weather conditions. The objectives of this study are the followings: (1) to reveal different contributing factors for SV and MV crashes with aggregate SPFs; (2) to identify the preferred modeling technique by comparing the Bayesian hierarchical Poisson model with correlated random effects with a Bayesian bivariate Poisson-lognormal model; and (3) to develop real-time crash risk evaluation models for SV and MV crashes while accounting for the

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seasonal variations, crash unit level and segment level unobserved heterogeneity.

For aggregate analyses, SV and MV crashes should be analyzed separately, as stated by Geedipally and Lord (2010). A previous study (Yu et al., 2013a) on the same freeway section concluded that a Bayesian hierarchical Poisson model with correlated random effects is appropriate for analyzing the SV and MV crashes simultaneously. In the current study, a Bayesian bivariate Poisson-lognormal model is introduced in addition to the hierarchical Poisson model with correlated random effects to model the SV and MV crashes. The difference between these two models' structures lies in the random errors for the MV and SV crash frequencies: for the hierarchical Poisson model, each segment shares the same random error; for the bivariate Poisson-lognormal model, two joint distributed random errors have been assigned to each segment. The two models were compared regarding the model fits and parameter estimations.

Moreover, disaggregate real-time crash risk evaluation models are estimated for the SV and MV crashes separately. Pande and Abdel-Aty (2006a) stated that it is important to analyze the crashes by type, particularly in real-time risk assessment. In this study, real-time crash risk evaluation models are estimated with multi-level Bayesian logistic regression models incorporating real-time traffic data, weather information and geometric characteristics. In addition to the basic logistic regression models, random parameters are introduced to account for the distinct seasonal effects highlighted in a previous study (Ahmed et al., 2012a). Furthermore, segment-level random effects are employed to account for the unobserved heterogeneity caused by various geometric characteristics. Moreover, crash-unit-level random parameter models are estimated to present the varying effects of different crash observations and their matched non-crash cases.

This paper is divided into five sections. First, previous studies related to SV and MV crashes and the relevant modeling techniques, such as Bayesian multivariate Poisson-lognormal and random parameter logit models, are discussed. The second section provides a brief description of the data preparation procedures, followed by a description of the methodologies used in this study. The fourth section presents the model results and a discussion about the estimated parameters and the model's goodness-of-fit. Finally a summary of the work is given.

2. Background

2.1. Aggregate analysis for crash types

Safety performance functions have been employed to analyze crash occurrence contributing factors. Identifying crash occurrence-related variables can be useful in improving traffic safety at both the planning and safety improvement re-design stages of transportation practices. However, there is a question of whether it is necessary to develop multiple distinct SPFs (for each crash type) instead of a unique one for total crashes (Mensah and Hauer, 1998). A variety of studies (Ivan et al., 2000; Qin et al., 2004; Ye et al., 2009; Geedipally and Lord, 2010) have proved from different aspects that it is beneficial to analyze SV and MV crashes separately while considering their correlation effects.

Ma and Kockelman (2006) utilized a multivariate Poisson model to simultaneously analyze crash counts with different injury severity levels through the Bayesian paradigm, providing a systematic approach to estimating correlated count data. Recently, more-advanced multivariate Poisson-lognormal (MVPLN) models have been adopted to analyze correlated count data. MVPLN models were argued to be superior to multivariate Poisson models because the capability of accounting for over-dispersion and its more general correlation structure allows for negative correlations. Several studies (Park and Lord, 2007; Ma et al., 2008; El-Basyouny and Sayed, 2009) have utilized multivariate Poisson-lognormal models to analyze crash frequency by severity.

In our previous study (Yu et al., 2013a), a Bayesian hierarchical Poisson model was introduced to simultaneously model the SV and MV crash frequencies. Correlated random effects were employed to handle the over-dispersion problem and to consider the shared unobserved heterogeneity for the two crash types within the same segment. In the current study, a simplified MVPLN model, the Bayesian bivariate Poisson-lognormal model, is used to analyze crash frequencies by crash types. The results are compared with the Bayesian hierarchical Poisson model with correlated random effects.

2.2. Real-time crash risk evaluation models

Real-time crash risk evaluation models were estimated to reveal crash occurrence precursors where the results could be utilized in traffic management systems. With advanced traffic surveillance systems (loop detectors, speed radars, automatic vehicle identification systems), traffic statuses prior to crash occurrence can be identified and matched with crash records. Various approaches have been adopted to develop the crash risk evaluation models, including matched case–control logistic regression (Abdel-Aty et al., 2004), neural network (Pande and Abdel-Aty, 2006a,b; Pande et al., 2011), Bayesian logistic regression (Ahmed and Abdel-Aty, 2012; Ahmed et al., 2012b) and support vector machine (Yu and Abdel-Aty, 2013a) models. Based on such crash risk assessment models, variable speed limit (VSL) systems have been developed and tested through simulation to evaluate the effectiveness of active traffic management (ATM) in improving traffic safety (Abdel-Aty et al., 2007).

2.3. Random parameter logit models

Random parameter models have become popular because their parameter estimations can vary across different levels, which is important for capturing unobserved heterogeneity. The random parameter logit model (or mixed logit model) has been widely utilized in crash injury severity analyses (Milton et al., 2008; Anastasopoulos and Mannering, 2011; Kim et al., 2012). Previous studies have demonstrated that random parameter models can account for unobserved effects (roadway characteristics, environmental factors and driver behavior). However, to the best of our knowledge the random parameter logit model has not yet been employed to estimate real-time crash risk evaluation models. In this study, random parameter logit models were introduced to develop real-time crash risk evaluation models while revealing the distinct seasonal effects and the various effects at the crash unit level.

2.4. Random effects logistic regression models

Another approach to account for the unobserved heterogeneity of the logit models is to develop hierarchical logit models. Huang et al. (2008) introduced hierarchical Bayesian binomial logistic regression models to perform multi-vehicle crash injury severity analysis. By incorporating the driver-vehicle units' correlations in the same multi-vehicle crashes, 28.9% of the unexplained variations resulting from between-crash variance were accounted for. Moreover, Yu et al. (2013b) employed hierarchical logistic regression models to analyze crash type propensity with segment-level random effects to account for the unobserved heterogeneity, and better classification accuracies were achieved with the additional random effects. In this study, the random effects were formulized

Table 1
Summary of variables' descriptive statistics for the aggregate analysis models.

Variables	Description	Mean	Std. dev.	Min	Max
Dependent variables					
Multi-vehicle crash frequency	Crash frequency counts for multi-vehicle crashes	3.87	4.67	0	21
Single-vehicle crash frequency	Crash frequency counts for single-vehicle crashes	5.49	6.66	0	38
Independent variables					
Degree of curvature	Degree of curve per segment	1.44	1.53	0	4.25
Curve length ratio	Percentage of curve length to total segment length	0.52	0.46	0	1.0
Median width		25.23	15.26	2	50
Speed limit		59.3	4.89	50	65
Three lane	1 If three-lane segment; 0 if two-lane segment	0.58	0.49	0	1.0
Grade	Longitudinal grade, eight categories: Upgrade: 0–2% = 1, 2–4% = 2, 4–6% = 3, 6–8% = 4; Downgrade: 0-(-2)%=5, (-2)-(-4)%=6, (-4)-(-6)%=7, (-6)-(-8)%=8	4.45	2.40	1	8
Exposure variables					
LogAADT	Logarithmic transformation of segment AADT	10.26	0.06	10.14	10.28
LogLength	Logarithmic transformation of segment length	−1.59	0.54	−2.38	−0.08

at the homogeneous segment level to account for the unobserved heterogeneity caused by the geometric characteristics.

3. Data preparation

The chosen freeway section starts at mile marker (MM) 205 and ends at MM 220. This stretch of road contains the 1.69-mile-long Eisenhower Memorial Tunnel (MM 213.18–MM 214.87). In addition to several sharp horizontal curves, the roadway section features longitudinal grades that vary from 1.3% to 7% (absolute values). The elevations in the studied area vary from 8,700 ft to more than 14,000 ft, with the highest peaks above the tunnel. Affected by the high altitudes, the climate (visibility, temperature and precipitation) can vary abruptly within this short distance. All these characteristics make this freeway section a challenging but interesting location for this traffic safety study.

3.1. Aggregate analysis

Five years of crash data (from 2006 to 2010) on I-70 in Colorado were used along with roadway geometry information to prepare the aggregate analyses dataset. A total of 1171 crashes were documented within the studied period, among which 487 were multi-vehicle crashes and the remaining 684 single-vehicle crashes. The 15-mile freeway section was split into 120 homogenous segments (60 in each direction) according to the major segmentation criterion of roadway alignment homogeneity and the Roadway Characteristics Inventory (RCI) data. Both horizontal and vertical alignments were scrutinized. A minimum-length of 0.1 mile was used to avoid the low exposure problem and the large statistical uncertainty of the crash rates in short segments (Ahmed et al., 2011). Table 1 provides descriptive statistics of the significant variables included in the final models. In the previous studies, daily vehicle miles traveled (VMT) was calculated by multiplying the segment lengths with the corresponding average annual daily traffic (AADT) to represent the segments' exposure. Because many previous studies have concluded that single-vehicle crashes are unrelated to high volumes, AADT and segment length were used separately to reflect different exposure measures for crash occurrence.

3.2. Disaggregate analysis

Four datasets were included in the disaggregate analysis: (1) crash data from Oct 2010 to Oct 2011 provided by the Colorado Department of Transportation (CDOT); (2) roadway geometric characteristic data from the RCI; (3) real-time weather data recorded by 6 weather stations along the studied roadway segment; and (4) real-time traffic data detected by 30 RTMS radars. A total of 259 crashes were documented and matched with real-time traffic and weather data. Of these crashes, 109 were multi-vehicle and 150 were single-vehicle.

Information about temperature, visibility and precipitation were recorded by the weather stations. The weather data were not recorded continuously; once weather condition changes reached a preset threshold, a new record was added to the archived data. Crashes were assigned to the nearest weather station according to the MM. For each specific crash, based on the reported crash time, the closest weather record prior to the crash time was extracted and used as the crash time weather condition.

RTMS radars archived speed, volume and occupancy information at 30-second intervals. The real-time traffic data that correspond to each crash were prepared by first aggregating the raw data into 5-min intervals (the 30-second raw data have random noise and are difficult to work with within a modeling framework). Traffic data 5–10 min prior to the crash times were selected to represent the traffic conditions. A previous study (Ahmed and Abdel-Aty, 2012) investigated the optimal traffic data aggregation level issue. In their study, 1-min speed data were aggregated to different levels (2, 3, 5, and 10 min), and it was concluded that the 5-min interval provided the best accuracy in the models. Moreover, the 5–10 min traffic variables prior to the reported crash time were extracted to avoid confusing pre- and post-crash conditions, as also used in many previous studies (Oh et al., 2001; Abdel-Aty and Pande, 2005; Yu and Abdel-Aty, 2013a). For each specific crash, information was collected from two upstream and two downstream RTMS detectors. For example, if a crash happened at 15:25 at MM 211.3, the corresponding traffic status within the time interval 15:15 and 15:20 recorded by upstream RTMS radars at MM211.8 (U1) and MM210.8 (U2) and downstream radars at MM213.3 (D1) and MM216.7 (D2) would be collected and matched with the crash. Fig. 1 shows the RTMS detector names and their relationship with the crash locations. For each observation, average, standard deviation and coefficient of variance values, the speed, occupancy and volume were calculated for the four detectors. Thus, there are 36 (3 traffic flow parameters × 3 measures × 4 detectors) explanatory variables for each observation. Moreover, the matched case–control design was adopted in this study to create a non-crash dataset. The matched case–control design is frequently utilized in disaggregate crash occurrence studies because the confounding

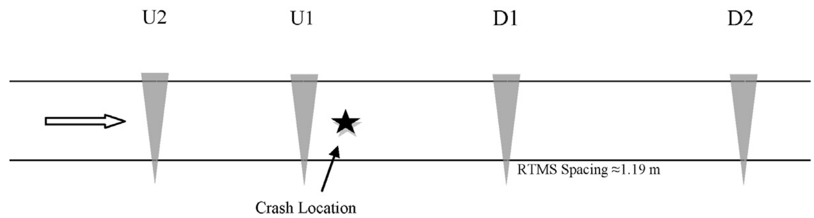

Fig. 1. Arrangements of RTMS detectors.

factors can be controlled for by matching (Breslow and Day, 1980). For each specific crash case, four non-crash cases were identified and matched. The non-crash cases were selected according to the following example procedure: if a crash happened on Tuesday (May 24, 2011), four non-crash cases will be selected for the exact same time two weeks before and two weeks after the crash time (May 10, May 17, May 31, and Jun 7), given that a crash did not occur.

Finally, in the multi-vehicle crash dataset, there are 109 crash cases matched with 429 non-crash cases, while for the single-vehicle crash dataset, there are 150 crash cases with 562 non-crash cases (due to detector malfunctions, not all crash cases were matched with exactly four non-crash cases). Tables 2 and 3 present the descriptive statistics for the significant variables included in the multi-vehicle and single-vehicle real-time crash risk evaluation models, respectively. Possible multicollinearity problems were checked in the preliminary analysis, which is not shown in the paper.

4. Methodology

To perform the multi-level crash type analyses, SPFs were first estimated with the Bayesian bivariate Poisson-lognormal formulation and the Bayesian hierarchical Poisson model with correlated random effects. Then, real-time crash risk evaluation models for multi-vehicle and single-vehicle crashes were estimated using the multilevel Bayesian logistic regression models, within which random parameters were utilized to capture the seasonal effects. The segment level random effects were employed to account for unobserved heterogeneity caused by geometric characteristics.

4.1. Bivariate Poisson-lognormal model

As concluded by previous studies utilizing multivariate Poisson-lognormal (MVPLN) models (Park and Lord, 2007; El-Basyouny and Sayed, 2009), these models are able to handle the over-dispersion issue and provide a more general correlation structure. The bivariate Poisson-lognormal (BPLN) model is a simplified MVPLN model. In the BPLN model, the crash frequency Y_{it} has a Poisson distribution conditional on the σ-field generated by the random variables of unobserved heterogeneity ε_1, ε_2 and the set of independent explanatory variables X_{it} (Munkin and Trivedi, 2002). The model can be set up as follows:

$$Y_{it} \sim \text{Poission}(\lambda_{it} \quad \text{for} \quad t = 1, 2)$$

$$\log \lambda_{it} = \log e_{it} + X_{it}\beta + \varepsilon_t$$

The random errors ε_1 and ε_2 are assumed jointly normally distributed

$$(\varepsilon_1, \varepsilon_2) \sim N\{(0, 0), (\sigma_1^2, \rho\sigma_1\sigma_2, \sigma_2^2)\}$$

where ρ is the correlation coefficient. Furthermore, the Bayesian hierarchical Poisson model with correlated random effects can be set up as:

$$Y_{it} \sim \text{Poission}(\lambda_{it} \quad \text{for} \quad t = 1, 2)$$

$$\log \lambda_{it} = \log e_{it} + X_{it}\beta + b_i$$

$$b_t \sim N(0, \sigma_b^2)$$

where the correlated random effects are set to follow normal distribution $b_i \sim N(0, 1/a)$, w here a is the precision parameter and is specified to be gamma prior as $a \sim$ Gamma (0.001, 0.001).

4.2. Multilevel logistic regression model

Suppose the crash occurrence has the outcomes $y = 1$ or $y = 0$ with respective probability p and $1 - p$. The multi-level logistic regression can be set up as follows:

$$y \sim \text{Binomial}(p)$$

$$\log it(p) = \log\left(\frac{p}{1-p}\right) = XB_{t[i]} + \alpha_{j[i]}$$

where β_0 is the intercept and X is the vector of the explanatory variables. For $t = 1,2$, $B_{t[i]}$ is the vector of random coefficients for the explanatory variables,

$$B_t \sim N(M_B, \sum\nolimits_B), \quad \text{for} \quad t = 1, \ldots, T,$$

where M_B represents the mean of the distribution of the coefficients and Σ_B is the covariance matrix representing the variation of the coefficients. t represents the two seasons ($t = 1$ for dry season and $t = 2$ for snow season) or stands for the crash unit (crash observation and their matched non-crash cases) index.

$j[i]$ indexes the segment where observation i occurs and $\alpha_{j[i]}$ is the random effects variable defined in the model, which represents the segment-specific random effects in this study:

$$\alpha_j \sim N(U_j\gamma, \sigma_\alpha^2), \quad \text{for} \quad j = 1, \ldots, 120,$$

where $\boldsymbol{U}$ is the matrix of segment-level predictors, γ is the vector of coefficients for the segment-level regression, and σ_α is the standard deviation of the unexplained segment-level errors.

4.3. Bayesian inference

Full Bayesian inference was employed in this study. The random effects (ε_t,b_t, and α_j in these models are unknown and thus have their own prior distribution, $p(\varnothing)$. The joint prior distribution is (Gelman et al., 2004)

$$p(\varnothing, \theta) = p(\varnothing)p(\theta|\varnothing),$$

Table 2
Summary of descriptive statistics of variables for the multi-vehicle crash model.

Variables	Description	Mean	Std. dev.	Min	Max
Crash	Binary index for crash occurrence (1 for crash and 0 for non-crash cases)	0.20	0.40	0	1
D1 Av. Spd.	D1 detector average speed (mph)	54.59	13.94	7	76.6
D2 Std. Occ.	D2 detector standard deviation of occupancy (%)	1.93	1.62	0	21.17
Visibility	Visibility (miles)	2.97	2.54	0	7.1

and the joint posterior distribution can be defined as

$$p(\varnothing, \theta|y) \propto p(\varnothing, \theta)p(y|, \varnothing\theta) = p(\varnothing, \theta)p(y|\theta).$$

For each model, three chains of 15,000 iterations were set up in WinBUGS (Lunn et al., 2000), and 5000 iterations were used in the burn-in step. Convergences of the models were checked by monitoring the MCMC trace plots for the model parameters: if all values were within a zone without strong periodicities or tendencies, the model was considered convergent. For the aggregate analysis models, DIC was selected as the evaluation measure. The DIC, recognized as a Bayesian generalization of AIC (Akaike information criterion), is a combination of the measure of model fitting and the effective number of parameters. A smaller DIC indicate a better model fitting. According to Spiegelhalter et al. (2003), differences greater than 10 can rule out the model with a higher DIC. Differences between 5 and 10 are considered substantial. For the Bayesian logistic regression models, areas under the receiver operating characteristic curves (AUC) were used to represent the classification abilities for different models.

5. Modeling results and discussions

This section discusses the modeling results of the safety performance functions for SV and MV crashes, followed by real-time crash risk evaluation models for the MV and SV crashes.

5.1. Aggregate analysis

In the aggregate analyses, the crash frequency per segment for MV and SV crashes were analyzed simultaneously while considering their correlation effects. Two models were considered: a Bayesian bivariate Poisson-lognormal model and a Bayesian hierarchical Poisson model with correlated random effects. Table 4 provides the parameter estimations, 95% confidence intervals and goodness-of-fit for both candidate models.

For the multi-vehicle crashes, the degree of curvature is significant with a negative sign, which indicates that segments with sharp curves are less likely to have crashes compared with flat curves. Similar results have been reported in previous studies (Shankar et al., 1995; Anastasopoulos et al., 2008). This result may be understood as drivers being more cautious when navigating sharp curves. The curve length ratio variable represents the percentage of curve length to the total segment length. This variable has a positive sign, which demonstrates that segments with longer curves are more likely to have crashes. The three-lane indicator is negatively associated with high crash frequency, which indicates that fewer crashes occurred at three-lane segments. The median width variable is also significant with a negative sign, which demonstrates that a larger median could most likely reduce crash occurrence. Moreover, the two exposure variables are both significant with positive signs, which can be understood as longer segments being likely to have more crashes, while a larger AADT may also increase the likelihood of crashes.

For the single-vehicle crashes, the three-lane indicator is again significant with a negative sign, which indicates that two-lane segments are likely to have a higher crash frequency for both SV and MV crashes. The median width variable is negatively related to the single-vehicle crashes, which indicates that narrow median segments increase the likelihood of single-vehicle crashes. The speed limit variable was found to be significant with a positive sign, indicating that with higher speed limits drivers travel at higher speeds, thus increasing the likelihood of single-vehicle crash occurrence. In addition, the longitudinal grade variables are significant (reference to the Grade [8], downgrade slopes range from 6% to 8%). Generally, the steeper the slope, the higher the crash risk, and segments with downgrade slopes are relatively more hazardous than the corresponding upgrades with the same slope ranges. Furthermore, segment length is the only significant exposure parameter, which demonstrates that single-vehicle crash occurrence is unrelated to high AADTs. This result is consistent with a previous study that concluded that single-vehicle crashes are more likely to happen at small volume-capacity ratios (Ivan et al., 2000).

For the model comparisons, DIC was chosen as the evaluation criterion for comparing the two models. The bivariate Poisson-lognormal model has a substantially smaller DIC value than the hierarchical Poisson model (more than 10), which indicates that the bivariate Poisson-lognormal model is superior to the hierarchical Poisson model. In addition to the goodness-of-fit, in the parameter estimation the degree of curvature and curve length ratio variables are not significant at the 95% level in the hierarchical Poisson model, as their credible intervals cross zero, while both are significant in the bivariate Poisson-lognormal model. Furthermore, although the correlated random effects in the hierarchical Poisson model are able to capture the shared unobserved heterogeneity, the correlation coefficient of the two count variables cannot be obtained. However, in the bivariate Poisson-lognormal model, it can be seen that the correlation coefficient is 0.68 for the SV and MV crashes, which demonstrates that these two crash frequency variables are highly correlated and that researchers should consider the correlation effects when analyzing these two crash conditions.

Table 3
Summary of descriptive statistics of variables for the single-vehicle crash model.

Variables	Description	Mean	Std. dev	Min	Max
Crash	Binary index for crash occurrence (1 for crash and 0 for non-crash cases)	0.21	0.41	0	1
D2 Av. Spd.	D2 detector average speed (mph)	55.64	11.63	5.77	77.45
D2 Log Vol.	D2 detector logarithmic transformation of volume (volume per 5 min)	1.93	1.62	0	21.17
D1 Std. Occ.	D1 detector standard deviation of occupancy (%)	1.81	1.59	0	22.23

Table 4
Parameter estimations and model goodness-of-fit for aggregate analysis.

Variable	Bivariate Poisson-lognormal				Hierarchical Poisson			
	Mean	Std	2.5%	97.5%	Mean	Std	2.5%	97.5%
Multivehicle								
Intercept	4.15	2.53	−0.46	8.6	−23.4	3.26	−28.3	−16.4
Degree of curvature	−0.21	0.09	−0.4	−0.02	−0.03	0.07	−0.17	0.11
Curve length ratio	0.67	0.27	0.14	1.21	0.17	0.19	−0.22	0.55
Three lane	−0.64	0.2	−1.05	−0.25	−0.85	0.19	−1.23	−0.46
LogLength	1.25	0.15	0.96	1.57	1.09	0.13	0.84	1.35
LogAADT	1.01	0.24	0.65	1.44	2.67	0.32	1.99	3.15
Median Width	−0.016	0.006	−0.029	−0.004	−0.012	0.005	−0.024	−0.0001
σ_{11}	0.22	0.11	0.06	0.48	N/A			
Single								
Intercept	1.69	0.75	0.32	3.23	2.4	0.73	0.98	3.84
Lane	−0.48	0.2	−0.86	−0.08	−0.38	0.17	−0.72	−0.024
Median Width	−0.016	0.006	−0.028	−0.004	−0.012	0.005	−0.023	−−0.001
Speed limit	0.04	0.012	0.014	0.06	0.025	0.011	0.002	0.046
Loglength	0.96	0.14	0.68	1.23	0.91	0.12	0.68	1.14
Grade [1]	−1.77	0.34	−2.43	−1.13	−1.49	0.31	−2.14	−0.88
Grade [2]	−0.52	0.23	−0.98	−0.08	−0.38	0.25	−0.88	0.08
Grade [3]	−0.56	0.16	−0.89	−0.24	−0.51	0.19	−0.94	−0.13
Grade [4]	−0.17	0.23	−0.63	0.29	−0.18	0.24	−0.66	0.29
Grade [5]	−1.58	0.31	−2.2	−1.0	−1.37	0.28	−1.92	−0.83
Grade [6]	−0.36	0.28	−0.92	0.18	−0.51	0.29	−1.04	0.06
Grade [7]	−0.4	0.25	−0.9	0.10	−0.28	0.24	−0.76	0.22
σ_{22}	0.48	0.12	0.28	0.74	N/A			
σ_{12}	0.22	0.12	0.12	0.46	N/A			
Correlation	0.68				N/A			
Dispersion parameter	N/A				0.28	0.06	0.17	0.42
DIC	1183.77				1195.41			

5.2. *Multi-vehicle crash risk evaluation models*

Four models were estimated to assess the real-time crash risks: (1) a Bayesian fixed parameter logistic regression model; (2) a Bayesian random parameter logistic regression model accounting for seasonal variations; (3) a Bayesian multi-level logistic regression model accounting for both the seasonal variations and unobserved segment-level heterogeneity; and (4) a Bayesian random parameter logistic regression account for crash level unobserved heterogeneity. Tables 5 and 6 show the parameter estimations and model fit for the four developed models.

Three variables were found to be significant in the multi-vehicle crash risk evaluation models: the average speed recorded by the D1 detector is significant at a 95% level with a negative sign, which indicates that congested conditions at downstream detectors would contribute to an increase in the likelihood of multi-vehicle crashes. The visibility variable is significant and proved to be negatively related to multi-vehicle crash occurrence, which can be understood as multi-vehicle crashes being more probable during poor visibility conditions. Car-following and lane-changing maneuvers are much more difficult under poor visibility conditions, which can lead to sideswipes or rear-end crashes. The standard deviation of occupancy of the D2 detector is found to be significant with a positive sign, which demonstrates that a turbulent area exists downstream, forcing the approaching vehicles to slow down. Drivers who are unable to reduce their speeds efficiently are prone to causing rear-end crashes.

As stated and proved in the previous work (Ahmed et al., 2011), significant seasonal effects exist on the chosen freeway segment. The snow season ranges from October to April, and the dry season begins in May and ends in September. Thus, we hypothesized that variable estimations may vary across the two seasons, making the Bayesian random parameter logistic regression model appropriate for estimation. As seen from the results, the average speeds recorded by the D1 detector are nearly identical for the two seasons. The visibility variable has distinct effects for crash occurrence in the two seasons: visibility is not significant during the dry season, while during the snow seasons it is significant with a negative sign. Moreover, the standard deviation of the occupancy of the D2 detector is significant for both seasons and has a greater effect on increasing the snow season's crash occurrence likelihood, with an odds ratio of 1.49, compared with the dry season, with an odds ratio of 1.25. The abovementioned findings indicate that employing the random parameters confers the benefits of capturing the seasonal variation effects.

Although the seasonal random parameter model was able to capture the distinct seasonal effects of the explanatory variables, the cause–effects of the crashes in each season were averaged. With the matched case–control design, each crash observation was matched with four non-crash cases. These five observations were considered as a crash unit, and, based on the crash unit level, random parameters were employed in the Bayesian logistic regression to account for the unobserved heterogeneity.

In addition to the seasonal variations, one more important aspect that needs to be considered is the segment variations. Despite the matched case–control design controls for the effects of geometric characteristics on crash occurrence, the geometric features may have effects on the selected traffic and weather variables. For example, roadway capacities would vary with the geometric features. To account for the unobserved segment level heterogeneity, the Bayesian multi-level logistic regression model was estimated. Median width and the three-lane indicator were found to significantly contribute to the segment variation effects.

For the model comparisons, AUC reflects the models' abilities to correctly classify the crash and non-crash cases and was chosen to be the evaluation criterion. The basic Bayesian fixed parameter logistic regression model provides the worst goodness-of-fit with an AUC of 0.75. Because the model accounts for more unobserved heterogeneity the better model fit achieved, the Bayesian multi-level logistic regression model has the best AUC of 0.78. However, the four models' AUC values are very comparable. The purpose of introducing seasonal random parameters, crash unit level random parameters and segment-level random effects to the basic Bayesian logistic regression model is not to improve the goodness-of-fit.

Table 5
Fixed parameter and seasonal random parameter model results.

	Fixed parameter model				Seasonal random parameter model			
Variables	Mean	Std.	2.5%	97.5%	Mean	Std.	2.5%	97.5%
D1 Av. Spd.	−0.032	0.004	−0.039	−0.025	−0.034 [Dry]	0.0066	−0.048	−0.022
					−0.032 [Snow]	0.0046	−0.041	−0.023
Visibility	−−0.159	0.049	−0.26	−0.064	−0.03 [Dry]	0.077	−0.18	0.12
					−0.26 [Snow]	0.072	−0.41	−0.12
D2 Std. Occ.	0.33	0.067	0.21	0.47	0.22 [Dry]	0.11	0.02	0.45
					0.40 [Snow]	0.08	0.23	0.57
ROC	0.75				0.76			
Number of observations	538				538			

Table 6
Multi-level model and crash-level random parameter model.

Variables	Crash unit level random parameter model				Multi-level model			
	Mean	Std.	2.5%	97.5%	Mean	Std.	2.5%	97.5%
D1 Av. Spd.	−0.032	0.004	−0.04	−0.02	−0.044 [Dry]	0.008	−0.06	−0.03
	(0.007)[a]	(0.004)	(0.002)	(0.018)	−0.043 [Snow]	0.006	−0.06	−0.03
Visibility	−0.168	0.054	−0.276	−0.065	−0.067 [Dry]	0.08	−0.23	0.09
	(0.062)	(0.042)	(0.013)	(0.168)	−0.36 [Snow]	0.08	−0.54	−0.2
D2 Std. Occ.	0.343	0.083	0.195	0.519	0.25 [Dry]	0.12	0.03	0.48
	(0.132)	(0.105)	(0.022)	(0.414)	0.41 [Snow]	0.09	0.24	0.59
Median width					0.02	0.009	0.002	0.04
Three lane					0.54	0.28	−0.02	1.09
Segment-level error					0.48	0.08	0.34	0.68
ROC	0.77				0.78			
Number of observations	538				538			

[a] Standard errors of the variance of the coefficients in parentheses.

Table 7
Fixed parameter and seasonal random parameter model results.

Variables	Fixed parameter model				Seasonal random parameter model			
	Mean	Std.	2.5%	97.5%	Mean	Std.	2.5%	97.5%
D2 Av. Spd.	−0.066	0.0073	−0.081	−0.052	−0.094 [Dry]	0.019	−0.14	−0.059
					−0.06 [Snow]	0.008	−0.076	−0.045
D1 Std. Occ.	0.21	0.065	0.089	0.34	0.29 [Dry]	0.18	−0.047	0.64
					0.20 [Snow]	0.07	0.072	0.35
D2 Log Vol.	0.37	0.081	0.21	0.53	0.62 [Dry]	0.21	0.23	1.06
					0.32 [Snow]	0.088	0.15	0.49
ROC	0.755				0.76			
Number of observations	712				712			

Rather, we want to investigate the seasonal effects on the selected parameters, the variations of different crash units and reveal the segment level variations' contributing factors.

5.3. Single-vehicle crash risk evaluation models

Similar to the multi-vehicle crash risk evaluation models, the same four models were developed for the single-vehicle crashes. Tables 7 and 8 present the results of the parameter estimations and models' fit. Three variables were found to be significantly associated with the single-vehicle crash occurrence. For the D2 detector, the average speed and logarithmic transformation of the 5-min volume are significant, which reflects slow moving traffic platoons at the downstream detector of the crash occurrence locations. Drivers traveling at high speeds from upstream approaching the slow moving traffic platoon have to reduce their speed in

Table 8
Multi-level model and crash-level random parameter model.

Variables	Crash-unit-level random parameter model				Multi-level model			
	Mean	Std.	2.5%	97.5%	Mean	Std.	2.5%	97.5%
D2 Av. Spd.	−0.062	0.008	−0.078	−0.045	−0.11 [Dry]	0.02	−0.15	−0.07
	(0.006)[a]	(0.003)	(0.003)	(0.013)	−0.07 [Snow]	0.009	−0.09	−0.06
D1 Std. Occ.	0.202	0.075	0.059	0.355	0.33 [Dry]	0.18	−0.03	0.69
	(0.095)	(0.064)	(0.023)	(0.272)	0.18 [Snow]	0.07	0.048	0.32
D2 Log Vol.	0.328	0.096	0.137	0.517	0.68 [Dry]	0.21	0.29	1.14
	(0.062)	(0.034)	(0.018)	(0.152)	0.41 [Snow]	0.09	0.22	0.61
Median width					0.015	0.007	0.006	0.03
Segment−level error				0.45	0.08	0.32	0.63	
ROC		0.77			0.77			
Number of observations		712			712			

[a] Standard errors of the variance of the coefficients in parentheses.

advance to avoid rear-end crashes. Quick braking is very hazardous, especially considering the steep slopes and that drivers may lose control of their vehicles, resulting in single-vehicle crashes. The standard deviation of the occupancy of the D1 detector is significant with a positive sign, which indicates large variations of occupancy downstream would increase the probability of single-vehicle crash occurrence. A Bayesian random parameter model was also estimated. It can be seen from the estimation results that all the variables are significant for both snow and dry seasons; the only differences detected are that the variables' impact effects on crash occurrence vary across the seasons.

Furthermore, considering the segment-level unobserved heterogeneity, a Bayesian multilevel logistic regression model was developed. Median width is the only significant variable that was found to contribute to the segment variations. Moreover, the crash-level random parameter Bayesian logistic regression model was estimated to represent the varying effects of different crash units.

For the model comparisons, identical results were obtained as those from the MV models: the fixed parameter model has the lowest AUC value of 0.755, while the multi-level model is the best, with an AUC of 0.77. Again, the AUCs are very comparable for the four developed models.

6. Conclusion

This paper presents a systematic multi-level analysis for single-vehicle and multi-vehicle crashes on a mountainous freeway. To provide a systematic approach for analyzing freeway crash data, this study utilized data from a 15-mile mountainous freeway section of I-70 in Colorado. Five years of crash data were analyzed in the aggregate studies. Due to data availability limitations, the disaggregate models were estimated based on one year of crash data along with real-time traffic and weather data. Previous studies found that that SV and MV crashes should be modeled separately both at the aggregate (Geedipally and Lord, 2010) and disaggregate (Pande and Abdel-Aty, 2006a) levels. In this study, the MV and SV crash data were analyzed separately for the safety performance functions and the real-time crash risk assessment models.

For the aggregate analyses, safety performance functions were estimated for the two crash types separately while considering their correlation effects. Two models were developed: (1) a Bayesian bivariate Poisson-lognormal model, a simplified MVPLN model that is often adopted to analyze crash frequencies for different crash injury severities, and (2) a Bayesian hierarchical Poisson model with correlated random effects accounting for over-dispersion and correlation issues. The MV crash occurrence was found to be related to the degrees of curvature, curve length ratios, lane numbers and median widths. The two exposure parameters (AADT and segment length) were both significant, which demonstrate that a higher AADT increases the probability of MV crash occurrence. The SV crashes were more associated with the median widths, speed limits, lane numbers and longitudinal grades. Only one exposure parameter (segment length) was significant, as SV crashes seem to be not related to high AADTs, which is consistent with previous studies. In addition, the Bayesian bivariate Poisson-lognormal model outperformed the Bayesian hierarchical Poisson model, with a substantially lower DIC value and two more significant variables. Moreover, the correlation coefficient for SV and MV crash counts is 0.68, which again indicates that these two crashes should be analyzed separately, though still considering the correlation effects.

In the disaggregate analyses, a traditional matched case–control design approach was employed to control for the impacts of geometric characteristics on crash occurrence. Bayesian logistic regression models were developed to capture the crash-prone traffic statuses. For the MV crashes, the average speed at the D1 detector and the standard deviation of occupancy at the D2 detector are significant along with the visibility conditions. For the SV crashes, the average speed and sum volume at the D2 detector and the standard deviation of occupancy at the D1 detector are significant.

Regarding the modeling approaches, the Bayesian random parameter models are capable of accounting for the seasonal variation effects and the varying crash-unit-level effects. Furthermore, the Bayesian multi-level models captured the unobserved heterogeneity caused by the geometric characteristics. Median widths and the number of lanes contributed to the segment-level variations in the MV crash model. For the SV crashes, median width is the only parameter found to significantly impact the segment variations. However, it is unacceptable to estimate a model with both crash unit level random parameters and segment level variations because the segment variations caused by geometric characteristics have already been accounted for by the crash-unit-level random parameters. The goodness-of-fit results for the four presented models are similar and comparable: the more complex the model, the better the model fit. Moreover, the purpose of introducing the seasonal random parameters, the crash-unit-level random parameters and the segment-level random effects to the basic Bayesian logistic regression model is not simply to improve the goodness-of-fit; we wanted to investigate the seasonal effects on the selected parameters and the variations of different crash units and to reveal the contributing factors of the segment level variations.

In addition to the abovementioned conclusions and methodological contributions of this study, the modeling results have substantial application potential. For the aggregate analysis results, because different crash occurrence contributing factors were identified for the MV and SV crashes, distinct sets of crash modification factors (CMFs) can be estimated for the two crash types. For example, improving the curvature design would have a positive effect on decreasing MV crashes, while lowering speed limits could alleviate SV crash occurrence frequency. Furthermore, the sophisticated real-time crash risk evaluation models are promising for use in ATM systems. Crash occurrence probabilities can be calculated in real time with on-line field data, and traffic management strategies such as VSL can be triggered when the risk reaches certain thresholds. In addition, because the SV and MV crashes have distinct crash hazard factors, freeway managers can employ different control strategies to reduce SV or MV crash risks or to balance the two crash risks and utilize the optimal control strategies. However, all these possible applications of the model results require further investigation, which could be addressed in future studies.

The results presented in this paper are based on the particular data from a mountainous freeway, which is somewhat unique. Further research with different data and infrastructure types are needed to confirm the results reported in this study. Moreover, as indicated in Yu and Abdel-Aty (2013b) that informative priors could improve the goodness-of-fit for the SPFs, the utilization of informative priors in the crash risk evaluation models can also be investigated.

References

Abdel-Aty, M., Pande, A., 2005. Identifying crash propensity using specific traffic speed conditions. Journal of Safety Research 36, 97–108.

Abdel-Aty, M., Pande, A., Lee, C., Gayah, V., Dos Santos, C., 2007. Crash risk assessment using intelligent transportation systems data and real-time intervention strategies to improve safety on freeways. Journal of Intelligent Transportation Systems 11, 107–120.

Abdel-Aty, M., Uddin, N., Pande, A., Abdalla, F., Hsia, L., 2004. Predicting freeway crashes from loop detector data by matched case–control logistic regression. Transportation Research Record: Journal of the Transportation Research Board 1897, 88–95.

Ahmed, M., Huang, H., Abdel-Aty, M., Guevara, B., 2011. Exploring a Bayesian hierarchical approach for developing safety performance functions for a mountainous freeway. Accident Analysis and Prevention 43, 1581–1589.

Ahmed, M., Abdel-Aty, M., Yu, R., 2012a. Assessment of the interaction between crash occurrence, mountainous freeway geometry, real-time weather and avi traffic data. Transportation Research Record: Journal of the Transportation Research Board 2280, 51–59.

Ahmed, M., Abdel-Aty, M., Yu, R., 2012b. A Bayesian updating approach for real-time safety evaluation using avi data. Transportation Research Record: Journal of the Transportation Research Board 2280, 60–67.

Ahmed, M., Abdel-Aty, M., 2012. The viability of using automatic vehicle identification data for real-time crash prediction. IEEE Transcations on Intelligent Transportation Systems 13, 459–468.

Anastasopoulos, P., Mannering, F., 2011. An empirical assessment of fixed and random parameter logit models using crash-and non-crash-specific injury data. Accident Analysis & Prevention 43, 1140–1147.

Anastasopoulos, P., Tarko, A., Mannering, F., 2008. Tobit analysis of vehicle accident rates on interstate highways. Accident Analysis & Prevention 40, 768–775.

El-Basyouny, K., Sayed, T., 2009. Collision prediction models using multivariate poisson-lognormal regression. Accident Analysis & Prevention 41, 820–828.

Geedipally, S., Lord, D., 2010. Investigating the effect of modeling single-vehicle and multi-vehicle crashes separately on confidence intervals of Poisson-gamma models. Accident Analysis & Prevention 42, 1273–1282.

Gelman, A., Carlin, J., Stern, H., Rubin, D., 2004. Bayesian Data Analysis. CRC Press, New York.

Huang, H., Chin, H., Haque, M., 2008. Severity of driver injury and vehicle damage in traffic crashes at intersections: a Bayesian hierarchical analysis. Accident Analysis & Prevention 40, 45–54.

Ivan, J., Wang, C., Bernardo, N., 2000. Explaining two-lane highway crash rates using land use and hourly exposure. Accident Analysis & Prevention 32, 787–795.

Kim, J., Ulfarsson, G., Kim, S., Shankar, V., 2013. Driver-injury severity in single-vehicle crashes in california: a mixed logit analysis of heterogeneity due to age and gender. Accident Analysis & Prevention 50, 1073–1081.

Lord, D., Mannering, F., 2010. The statistical analysis of crash-frequency data: a review and assessment of methodological alternatives. Transportation Research Part A 44, 291–305.

Lunn, D., Thomas, A., Best, N., Spiegelhalter, D., 2000. Winbugs-a Bayesian modelling framework: concepts, structure, and extensibility. Statistics and Computing 10, 325–337.

Ma, J., Kockelman, K., 2006. Bayesian multivariate Poisson regression for models of injury count, by severity. Transportation Research Record: Journal of the Transportation Research Board 1950, 24–34.

Ma, J., Kockelman, K., Damien, P., 2008. A multivariate Poisson-lognormal regression model for prediction of crash counts by severity, using Bayesian methods. Accident Analysis & Prevention 40, 964–975.

Mensah, A., Hauer, E., 1998. Two problems of averaging arising in the estimation of the relationship between accidents and traffic flow. Transportation Research Record: Journal of the Transportation Research Board 1635, 37–43.

Milton, J., Shankar, V., Mannering, F., 2008. Highway accident severities and the mixed logit model: an exploratory empirical analysis. Accident Analysis & Prevention 40, 260–266.

Munkin, M., Trivedi, P., 2002. Simulated maximum likelihood estimation of multivariate mixed-poisson regression models, with application. Econometrics Journal 2, 29–48.

Oh, C., Oh, J., Ritchie, S., Change, M., 2001. Real time estimation of freeway accident likelihood. In: Presented at the 80th Transportation Research Board Annual Meeting, Washington, DC.

Pande, A., Abdel-Aty, M., 2006a. Assessment of freeway traffic parameters leading to lane-change related collisions. Accident Analysis & Prevention 38, 936–948.

Pande, A., Abdel-Aty, M., 2006b. Comprehensive analysis of the relationship between real-time traffic surveillance data and rear-end crashes on freeways. Transportation Research Record: Journal of the Transportation Research Board 1953, 31–40.

Pande, A., Das, A., Abdel-Aty, M., Hassan, H., 2011. Real-time crash risk estimation: are all freeways created equal? Transportation Research Record. Journal of the Transportation Research Board 2237, 60–66.

Park, E.S., Lord, D., 2007. Multivariate poisson-lognormal models for jointly modeling crash frequency by severity. Transportation Research Record: Journal of the Transportation Research Board 2019, 1–6.

Qin, X., Ivan, J., Ravishanker, N., 2004. Selecting exposure measures in crash rate prediction for two-lane highway segments. Accident Analysis & Prevention 36, 183–191.

Shankar, V., Mannering, F., Barfield, W., 1995. Effect of roadway geometrics and environmental factors on rural accident frequencies. Accident Analysis & Prevention 27, 371–389.

Spiegelhalter, D., Thomas, A., Best, N., Lunn, D., 2003. Winbugs User Manual. MRC Biostatistics Unit, Cambridge.

Ye, X., Pendyala, R., Washington, S., Konduri, K., Oh, J., 2009. A simultaneous equations model of crash frequency by collision type for rural intersections. Safety Science 47, 443–452.

Yu, R., Abdel-Aty, M., Ahmed, M., 2013a. Bayesian random effect models incorporating real-time weather and traffic data to investigate mountainous freeway hazardous factors. Accident Analysis & Prevention 50, 371–376.

Yu, R., Abdel-Aty, M., Ahmed, M., Wang, X., 2013b. Crash type propensity analysis with Bayesian models using microscopic traffic and weather data. Compendium of papers. In: Transportation Research Board 2013 Annual Meeting, Washington, D.C.

Yu, R., Abdel-Aty, M., 2013a. Utilizing support vector machine in real-time crash risk evaluation. Accident Analysis & Prevention 51, 252–259.

Yu, R., Abdel-Aty, M., 2013b. Investigating different approaches to develop informative priors in hierarchical Bayesian safety performance functions. Accident Analysis & Prevention 56, 51–58.